Open Source, Dual Licensing and Software Competition

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Abstract

To distribute software, commercial firms have the opportunity to use some dual licensing strategy i.e. to provide their software under two different licensing terms (proprietary and open source). In this paper, we investigate the relevance and impacts of such distribution strategy in the presence of an already existing open source software. In this competitive setting, we determine in which conditions this strategy may be profitable for the commercial firm but also analyses the impact of such strategy on “traditional” open source communities and users.

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1 Introduction

We consider in this paper a situation where a commercial software editor competes with an Open Source Software (OSS) provider. To increase its sales/profits, we design a model in which the editor may introduce an OS version of its software through dual licensing. Doing so, he may capture part of OSS existing users. Conversely, this may also decrease its sales through the regular (proprietary) channel. We discuss this alternative distribution strategy through a theoretical modeling. We first suppose that two software are active: a proprietary software the code of which is protected by the editor and an Open Source (further coined OS) platform released under a GPL license and is produced by an Open Source community of developers. The market is covered or not by these two platforms. The editor chooses to test a new strategy to increase its profit by considering the use of a dual license which would allow him to provide a free version close to the commercial version under an open license. The interest of this strategy is to allow interactions between this open source version of the proprietary software (OSP) and the OS platform, through the action of developers. The development of the two OS software

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(OS and OSP) then uses one part of the code of the other, according the degree of openness of the OSP platform. The advantages of the development of the OSP platform can produce development spillovers on the proprietary platform. We analyze in this paper the profitability of this strategy which generates both positive and negative effect on the editor optimal fees and share of market.

The benchmark version of the model, without dual license, is developed in section 2 with the main result that it can provide. Then, section 3 introduces the dual license (DL) and analyses its conditions of profitability. Section 4 considers the possible transitions from a market without DL to a market with DL. This part of the paper considers the conditions of an improvement of the profit of the editor, the consequences on the fees and of the share of market of the proprietary version. It also considers the welfare issues. Section 5 concludes.

2 The Model

We consider the competition between platform software. A platform software is a software that developers use to build their own pieces of software. There are initially two platforms. One is released under a GPL license and is produced by an Open Source community of developers. The other (platform) software is produced by a commercial (i.e. profit motivated) editor.

2.1 The software editor

In the benchmark case, the commercial editor only distributes this software under a proprietary license terms (i.e. closed source code and not-free software, further coined P) and sells it at price \( p \) to form its revenues. With Dual Licensing (DL), it has the opportunity to distribute an additional version of its software under an open-source distribution. To keep things simple, let us assume that the original software is produced without cost\(^1\). It is reasonable to suppose that delivering the same software under different license terms does not imply specific costs\(^2\). Thus, using the DL does not generate additional costs. The editor maximizes its profit with respect to the license cost of the proprietary software and given the above simplifications, its profit \( \pi \) is simply defined by its revenues. Thus, \( \pi(p) = p m_p \) (where \( m_p \) denotes the number of adopters of the proprietary software).

2.2 User-Developers

There exists \( m \) potential software users. Without loss of generality, the total number of users is normalized to 1. Users have specific and different needs and any type software cannot fill any of these needs. Such heterogeneity requires for developers to create additional pieces of code so as to adapt the original software to its needs.

To depict this type of heterogeneity, let us assume that users are uniformly distributed on a unitary segment. The additional pieces of code require a development effort that we

\(^1\)Since software production costs are essentially fixed costs, the introduction of such costs would not qualitatively alter the results.

\(^2\)The only cost would be the distribution cost that is negligible.
can depict through an additional cost which increases with the distance to the location of the software on the segment. By convention, let us assume that the proprietary software is located at location 0 on the unitary segment while the OS software is located at location 1, opposite to the proprietary software. We can thus simply write the utility \( U_p \) of the proprietary software derived by the developer \( i \) when using the proprietary software by \( U_p(i) = v - \alpha i - p \) where \( v \), \( \alpha \) and \( p \) depict respectively the benefits derived from the P software, the additional development cost \((\alpha > 0)\) associated with this software, function of the location of user \( i \), and the fees payed to the firm by the proprietary software users.

Similarly to the P software, users also need to write additional lines of code and develop new pieces of software when they adopt the OS software. Because it is Open Source, some of its development effort may be shared among the developers inside the OS community and it is reasonable to assume the development effort be less costly. The utility \( U_{os} \) derived from OS adoption is then \( U_{os}(i) = u - \beta (1 - i) \) where \( u \) depicts the benefits derived from the OS software and where \( \beta (1 - i) \) is the additional development cost \((\beta > 0)\).

### 2.3 Outcomes in the Benchmark Case

The firm plays first and selects a licensing and price strategy. Observing software licensing terms and conditions, potential users play second and choose to adopt (or not) one among the different available software. The game is solved by backward induction. Since we are interesting in initially competitive outcomes, we focus on situations where the two software (OS, P) are active before the firm can choose to dual license. Depending on parameters, two situations can occur in the benchmark case.

In the first situation, all users initially adopt one software so that users divide among the P and the OS platform (see Appendix 1 for the computation of this equilibrium). This equilibrium is depicted by Lemma 1.

**Lemma 1.** [P-OS] Initial Full Adoption. If \( u - \beta \leq v \leq u + 2\alpha - \beta \) and \( u(2\alpha + \beta) + \beta v \geq \beta(2\alpha + \beta) \), then all user-developers adopt one of the two platforms. The commercial platform is sold at price \( p^* = (v + \beta - u)/2 \), the optimal profit of the commercial editor is equal to \( \pi^* = (v + \beta - u)^2/4(\alpha + \beta) \) and his share of market \( i^* = (v + \beta - u)/2(\alpha + \beta) \)

*Proof:* see Appendix.

The restrictions on parameters depict from one hand the limitations imposed by the coexistence of the two technologies. The intrinsic utility derived of the use of the proprietary software must be sufficiently high to allow its adoption by users corresponding exactly to its basic specification \((u - \beta \leq v)\). This utility must however be sufficiently small to allow the adoption of the OS software for the users corresponding exactly to the specification of this last \( v \leq u + 2\alpha - \beta \). The restriction \( u(2\alpha + \beta) + \beta v \geq \beta(2\alpha + \beta) \)

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3OS projects historically developed in reaction to proprietary standards because closed-source software were targeted to some special needs and could not able all developers’ needs. The two platforms are then located opposite to each other so as to reflect this complementarity.
attests from the other hand that the agent obtaining the smaller utility from the two software, \textit{i.e.} the agent indifferent between the two software, however prefers using one of them than non adopting.

In the second situation, some users initially adopt one software (P or OS) but also some does not adopt any of the two software since they do not obtain a positive utility with one of the two software (see Appendix 1 for the computation of this equilibrium). This equilibrium is depicted by Lemma 2.

**Lemma 2.** [P-⊘ -OS] Initial Partial Adoption. If \( v < 2\alpha \) and \( u(2\alpha + \beta) + \beta v < \beta(2\alpha + \beta) \), then a fraction \( m_p^* = v/2\alpha \) of all potential users adopts the proprietary software, a fraction \( m_{os} = u/\beta \) and the remaining part does not adopt any software. The commercial platform is sold at price \( p^* = v/2 \) and the optimal profit of the firm is then \( \pi^* = v^2/4\alpha \).

\textit{Proof}: see Appendix.

The intuition for these conditions are as follows. For both software, the development costs (respectively measured by \( \alpha \) and \( \beta \) has to be sufficiently high compared to the utility of the two software (respectively \( u \) and \( v \)). Otherwise, whenever these two costs are low relatively, all users would get a positive utility by adopting one of the two software. Note however that the condition on \( \alpha \) depends only on \( u \) while the condition on \( \beta \) depends on all parameters, including \( u \) and \( \alpha \). This asymmetry reflects another asymmetry between the proprietary software which determines its share of market in maximizing its profit while the OS share of market is finally dependent on the decisions and capacities of its proprietary competitor. The optimal level of the fees of the proprietary software depends only on its own intrinsic utility since the market of proprietary software is only limited in this case by the “reservation utility” of the non-adopters. The level of \( \alpha \) is an additional parameter determining the rate of profit: higher is \( \alpha \), sharper is the decrease of utility of the proprietary software users when \( i \) increases and smaller are the share of market and the profit.

When considering the two sets of parameters defining the two outcomes, we can check that these sets are incompatible. This rules out multiple equilibria in the benchmark case.

### 3 Equilibria with DL

The firm may introduce dual licensing if it is profitable to do so. Let us coin this hybrid platform as OSP (as it mixes some characteristics of both OS and P software). The introduction of the OSP software has several effects. First, the adopters of the OSP software may benefit from the functionalities \( (v) \) developed for the proprietary software without paying the license cost attached to the latter. Similarly to the other software, they incur an additional development cost depending on their location on the segment \( \alpha i \). Second, associated with the distribution of the OSP, some users developing on the OSP software will create new lines of codes of software. Unlike under the proprietary license terms,
these lines of codes will inherit from the properties of the OS project and the firm will benefit from these to improve its proprietary software. Thus, this generates a spillover from the OSP to the P platform. Third, the OS and OSP may benefit from each other through a decrease in the development cost required when adopting these platforms. Indeed, part of the development effort of OS users may be used by OSP users to develop more efficiently and conversely. Yet, the magnitude of this last effect crucially depends on the compatibility degree between the two platforms. The more the two platforms are compatible, the easier will be the exchange of ideas and code across the two platforms. On the opposite side, if the two platforms are completely incompatible, no exchange across platforms will be possible. Let us capture the compatibility degree by parameter $\lambda$ (with $0 < \lambda < 1$).

Introducing these three elements and supposing $a > 0$, we can reformulate the three utilities as follows:

$$
U_p(i) = v - \alpha i + am_{osp} - p \\
U_{osp}(i) = v - \alpha(1 - \lambda)i \\
U_{os}(i) = u - \beta(1 - \lambda)(1 - i)
$$

where $m_{osp}$ figures the proportion of the potential users adopting the hybrid platform. The term $am_{osp}$ then represents the amount of the spillovers exerted from the OSP platform on the proprietary one which increase the utility of proprietary software users.

The introduction of OS translates into new potential situations. Again, we concentrate on some parameter configurations where OS and P are both active in the benchmark case (without DL). Several situations are now possible but in all cases, the lemma (3) applies:

**Lemma 3.** When the OSP platform is active, the population adopting this platform is always adjacent to the population adopting the proprietary software.

*Proof*: see Appendix.

In the first case, the three platforms cover market. The conditions on parameters are defined in proposition (1):

**Proposition 1.** [P-OSP-OS] For the conditions of parameters given by

$$
u \leq v \text{ and } \begin{cases} \beta \leq u/(1 - \lambda) \\
or \\
\beta > u/(1 - \lambda) \text{ and } (v - u)/(1 - \lambda) \leq \alpha \leq v\beta / (\beta(1 - \lambda) - u) \end{cases}$$

or

$$
u > v \text{ and } \begin{cases} (u - v)/(1 - \lambda) < \beta \leq u/(1 - \lambda) \\
or \\
\beta > u/(1 - \lambda) \text{ and } \alpha \leq v\beta / (\beta(1 - \lambda) - u) \end{cases}$$
an equilibrium situation exists where some users adopt the OS platform, some adopt the OSP platform and remaining users adopt the OS platform. In that case, the distribution between users is as follows: 

\[ m^*_p = a(v - u + \beta(1 - \lambda))/2(\alpha + \beta)(1 - \lambda)(a + \alpha \lambda); \]

\[ m^*_{osp} = (u - v + \alpha(1 - \lambda))/(\alpha + \beta)(1 - \lambda) \quad \text{and} \quad m^*_{os} = (a + 2\alpha \lambda)(v - u + \beta(1 - \lambda))/2(\alpha + \beta)(1 - \lambda)(a + \alpha \lambda) \]

and the optimal price for the P platform is 

\[ p^* = a(v - u + \beta(1 - \lambda))/2(\alpha + \beta)(1 - \lambda) \]

Proof: see Appendix.

This depicts an equilibrium with full adoption where all users adopt one platform and each platform is adopted by at least one user. The conditions of adoption can be commented. We first observe that these conditions impose few restrictions on the relative values of \( u \) and \( v \), the intrinsic utilities of the proprietary and OS software. They however restrict the acceptable values of \( \alpha \) and \( \beta \). If the proprietary software has an advantage on the OS software concerning the intrinsic utility, there are two possible sets of coexistence for the three software. The first concerns only \( \beta \), i.e. the parameters which captures the shift of the utility of the OS when the user according to the location of the user. This decrease must be not too sharp to allow the coexistence, given the externalities of development coming for the OS software from the OSP platform. When \( \beta \) increases, another condition must be added: the advantage of the proprietary software can be compensated if, given \( \lambda \), the depreciation of the advantage of the OS remains small when the user moves from its ideal location at \( i = 1 \). When this depreciation increased, the coexistence imposes, still given \( \lambda \), that the depreciation of the utility of the proprietary software would be also rather hight. Quite symmetric conditions are observed when the intrinsic utility of the OS software is greater than the intrinsic utility of the proprietary one. The equilibrium level of the fees of the proprietary software increases, as intuitively expected, with the relative advantage of the intrinsic utility of the proprietary software. It also increases according to the level of the development spillovers from the OSP platform to the proprietary one. It depends also, but in a sense depending of the extrinsic utilities, on the degree of compatibility \( \lambda \) (when the proprietary software has an intrinsic advantage over the OS one, increasing the compatibility tends to increase the amount of the fees (which reveals easier environment for the proprietary software) whereas a decrease of the fees is the consequence of an increase of the compatibility when the OS software has an intrinsic advantage over the proprietary one.

Another case is also possible, where the three platforms are active but some users do not adopt. This case is delimited by proposition (2):

**Proposition 2.** \([P-OSP-\emptyset-OS]\) When \( v < \alpha(1 - \lambda) \) and \( \beta > \frac{nu}{u + \alpha(\lambda - 1)} \), an equilibrium situation exists when some users adopt the OS platform, some adopt the OSP platform, some adopt the OS platform while some users do not adopt any platform. In that case, the distribution between users is as follows: 

\[ m^*_p = av/(2\alpha(\lambda - 1)(a + \alpha \lambda)); \]

\[ m^*_{osp} = \frac{v(u + 2\alpha \lambda)}{2\alpha(\lambda - 1)(a + \alpha \lambda)} \] and \( m^*_{os} = u/\beta(1 - \lambda) \) and the optimal price for the P platform is 

\[ p^* = av/2\alpha(\lambda - 1) \] (proof: see Appendix)

Proof: see Appendix.

The last possibility emerges when the OSP platform serves as a means to crowd out the OS platform. This case is delimited by proposition (3).
**Proposition 3. [P-OSP]** If \( v < \alpha(1 - \lambda) \) and \( \beta > \frac{\mu_0}{v + \alpha(\lambda - 1)} \), an equilibrium situation exists when some users adopt the P platform \((m_p^* = a/2(a + \alpha \lambda))\) and some adopt the OSP platform \((m_{osp}^* = 1 - m_p^* = 1 - a/2(a + \alpha \lambda))\). In that case, the introduction of the OSP platform induces the exit of the OS platform \((m_{os}^* = 0)\). The P software is sold at price \( p^* = a/2 \) and the profit of the firm is \( \pi^* = a^2/4(a + \alpha \lambda) \) (proof: see Appendix).

**Proof**: see Appendix.

When \( \beta \) is sufficiently large, i.e. when the utility of the OS software decreases rapidly as users move away from their ideal location, the adoption of the OS platform is no longer efficient for users that switch to the OSP platform. It is thus possible to switch from a P-OS or from a P-OSP equilibrium to a P-OSP equilibrium. Interestingly, the price charged by the firm is here only dependent on the size on the externalities coming from the OSP. Everything else equal, when spillovers are large, the firm can charge a higher price to its direct customers (users of the P platform). In particular, the price does not depend on \( v \) since the amount of utility \( v \) is common to the two software. A practical implication of that is that the main driver of the price of the P-software is not the intrinsic value of the software but the amount of externalities generated by the OSP software.

When considering the two sets of parameters defining the three outcomes, we can check that these sets are all mutually incompatible. This rules out multiple equilibria also in the case with hybrid software.

## 4 DL decision

We here analyze the decision of the firm towards DL and its effect on market structure. Considering the potential equilibria of the game (in the benchmark case and with DL), we need to consider all the potential switches raised by the introduction of the DL licence. By comparing the different equilibrium conditions and check whether these are compatible, we can demonstrate that some switchers are not possible as revealed by the following table:

<table>
<thead>
<tr>
<th>From ...</th>
<th>P - OS</th>
<th>P - Rien - OSP</th>
</tr>
</thead>
<tbody>
<tr>
<td>To ...</td>
<td>P - OSP</td>
<td>Possible (conditions sup)</td>
</tr>
<tr>
<td></td>
<td>P - OSP - OS</td>
<td>Impossible</td>
</tr>
<tr>
<td></td>
<td>P - OSP - Rien - OS</td>
<td>Possible (conditions sup)</td>
</tr>
</tbody>
</table>

Figure 1: Potential switches induced by the introduction of DL.
Clearly, this means that a situation where all users adopt one software (full adoption) without DL cannot lead to a situation with partial adoption when the DL is introduced. All other possible shifts are possible.

4.1 P-OSP case

Let us first consider that the outcome with DL is of type P-OSP. This case raises the problem of the value of $\lambda$ to consider. Suppose that the possibilities are limited to $\lambda = 0$ (no compatibility) and $\lambda = \bar{\lambda} > 0$ (partial compatibility). The proprietary software can decide one value or the other before the OS has been crowd out. After this, the possibility progressively reduces to $\lambda = 0$ since the OS software does not exist and that there is nothing to gain from “opening”. Let us assume that we are in this long term case where $\lambda = 0$.

4.1.1 Initial Full Adoption

Let us first consider the case of initial full adoption (i.e. P-OS equilibrium), the firm has an incentive to dual license its software only when

$$\pi_{P-OSP}^* - \pi_{P-OS}^* > 0.$$ 

This condition is more restrictive than the condition of existence for this equilibrium ($v > \alpha$, $\alpha > 0$, and $u + \alpha < v$). This means that when the market is initially covered, implementing the DL strategy in that situation is not always profitable for the firm. As a corollary, excluding the OS platform through the introduction of DL is not always an optimal strategy and an “accommodating strategy” should be used instead.

When the market is initially fully covered and when it is profitable to DL, the introduction of a DL can or not finally crowd out the OS software. In all cases, it is interesting to compare the ways the introduction of the DL increases the profit of the proprietary editor. Proposition 5 proves that the result is not indeterminate.

Proposition 5. When introducing the DL is profitable for the firm (increases its profit), this introduction always decreases its market share and increases the amount of the fees. The conditions of profitability are given by the following inequalities:
Let us now consider the case $P_-$.

### 4.1.2 Initial Partial Adoption

**Proof**: see Appendix.

This proposition has some practical implications. It implies that the OSP acts as a substitute for both OS and P. In other terms, when it is profitable to DL, the market share of the P platform should always decrease. Despite this decrease, DL is profitable thanks to the spillover coming from the OSP. In turn, these spillovers allow the firm to better price its product even if the number of regular customers is reduced. The increase of quality of the proprietary software allows the users with a good location to pay more to obtain the proprietary version.

We have analyzed the consequence of the introduction of the OSP version from a welfare point of view. Our results are ambiguous (see Appendix). In some cases, the introduction of a DL increases the welfare in other cases not. The reason is that the increase of the utility of the P software users due to the spillover form the OSP and the resulting increase of the fees have the consequence to improve the way the editor of the proprietary software extracts the consumer surplus. In some cases, the decrease of the surplus of the proprietary software consumers is larger than the increase of the surplus of the OS and OSP users.

### 4.1.2 Initial Partial Adoption

Let us now consider the case $P_\ominus$-OS (partial adoption) as the initial case. It is profitable for the firm to implement a DL strategy when $\pi^*_P - \pi^*_{P_\ominus-OS}$. This translates into the following condition (for any $\lambda$):

\[
\left( v > 0\&\& \frac{\sqrt{v}}{v} < \alpha + \sqrt{\frac{2a}{2a+\beta}} > 0\&\& \frac{2a}{2a+\beta} + \beta > 0\&\& u > 0\&\& u + \alpha < v + \sqrt{\frac{2a}{2a+\beta}} > 0\&\& (v > \alpha \&\& v > 2\alpha + \beta \&\& 2\alpha + \beta \&\& u + \frac{\sqrt{2a}}{2a+\beta} > \beta) \right)
\]

\[
(v > 2\alpha + \beta \&\& u + 2\alpha + \beta > v) \&\& ((\sqrt{2a} > \beta \&\& \beta > 0) \&\& \sqrt{\beta} > 5\alpha + \beta)
\]

When considering that $\lambda = 0$, these conditions reduce to:

\[
\left( v < 2\alpha + \beta \&\& 2v > \alpha + \sqrt{\frac{2a}{2a+\beta}} > 0\&\& \frac{2a}{2a+\beta} + \beta > 0\&\& u + \alpha < v \&\& (\beta > \frac{2a(v+\alpha)}{v-2a}) \&\& (v > \frac{2a(v+\alpha)}{v-2a}) \&\& 2\alpha + \beta < 2\beta \&\& \sqrt{\beta} \leq \frac{2a(v+\alpha)}{v-2a} \&\& \left( v + \sqrt{2a} > 2\alpha \&\& \alpha > \frac{\sqrt{2a}}{\alpha}\right) \&\& \left( v + \sqrt{2a} > 2\alpha \&\& \alpha > \frac{\sqrt{2a}}{\alpha}\right) \right)
\]

\[
\left( \frac{2v}{1+\sqrt{2a}} < \alpha < v \&\& \left( 0 < \beta < \frac{2a(v+\alpha)}{v-2a} \&\& \frac{\sqrt{2a}}{\alpha} < 2\beta \&\& \left( v + \sqrt{2a} < 2\alpha \&\& \alpha > \frac{\sqrt{2a}}{\alpha}\right) \&\& \left( v + \sqrt{2a} < 2\alpha \&\& \alpha > \frac{\sqrt{2a}}{\alpha}\right) \right) \right)
\]
These conditions are rather hard to comment. Fortunately, we obtain also in this case very simple results concerning the evolution of the price and shares of market. They are summarized in Proposition (6) and its corollary:

**Proposition 6.** When the market is initially not fully covered, the market share of the P software with DL is always less than that without DL.

**Corollary 1.** When the market is initially not fully covered and when the firm finds it profitable to dual licence, the price of the P software is always higher than that without DL.

This is in light with the previous proposition: introducing the OSP in that case always lead to less important marketshare for the P software.

5 Conclusion

The topic of this paper has been to analyze the conditions in which proprietary software editors may find profitable to use dual licenses, to provide their software under two different licensing terms (proprietary and OS). We have investigated the relevance and impacts of such distribution strategy in the presence of an already existing open source software. In this competitive setting, we determined in which conditions this strategy may be profitable for the commercial firm but also analyzed the impact of such strategy on “traditional” open source communities and users. We found that the cases in which dual licensing is the good solution are sufficiently representative to be considered as a relevant strategy for editors. They can emerge as well in full covered markets (when the proprietary and OS software) cover all the needs than in markets not covered (when there remains potential users not convinced by the OS software quality and by the fees of the proprietary software). When a dual license is implemented, it has always the consequence to restrict the share of market of the commercial version (of the proprietary software) and to increase the fees. Different scenarios of transition are possible from the initial situation without dual license to the new one with this license. We found however that the result of the implementation of the dual license does not always improve the users welfare.

References


Appendix

Appendix 1: Computation of the benchmark case

Equilibrium of type P/OS

The user (noted \( i_{p/os} \)) indifferent between the two software is defined by \( U_p(i_{p/os}) = U_{os}(i_{p/os}) \). Hence, \( i_{p/os} = (v + \beta - u - p)/(\alpha + \beta) \). From that, we deduce the expression of the profit \( \pi = p \cdot i_{p/os} \) and obtain from the FOC that \( p^* = (v + \beta - u)/2 \) and \( m_p^* = (v + \beta - u)/2(\alpha + \beta) \). The optimal profit of the firm is then \( \pi^* = (v + \beta - u)^2/4(\alpha + \beta) \). Putting together the boundary condition for \( i_{p/os}(0 < i_{p/os} < 1) \), the condition for the utility of all users to be strictly positive (\( U_p(i_{p/os}) > 0 \)) and the second-order condition, we obtain the following existing conditions for this equilibrium : \( u < v + \beta \) and \( v \leq 2\alpha + \beta \) and \( u + v\beta/(2\alpha + \beta) > \beta \) or if \( u < v + \beta \) and \( v > 2\alpha + \beta \) and \( u + 2\alpha + \beta > v \). From these conditions, we can deduce the expression of P users’ surplus \( W_p = \int_{i_{p/os}}^{i_{p/os}} U_p(i)di = \frac{(v+\beta-u)(3\alpha+2\beta+2v\beta-3\alpha\beta-2\beta^2)}{8(\alpha+\beta)^2} \), of OS users’ surplus \( W_{os} = \int_{i_{p/os}}^{1} U_{os}(i)di = \frac{(v+\beta-u)(u(4\alpha+5\beta)-\beta(v+\beta))}{8(\alpha+\beta)^2} \), the total surplus of all users \( W_{users} = W_p + W_{os} = \frac{\pi^*}{8(\alpha+\beta)^2} + \frac{(v+\beta-u)(5u+3v-\beta)}{8(\alpha+\beta)^2} \) and the total welfare \( W \) including the profit of the firm \( W = W_{users} + \pi^*/8(\alpha+\beta)^2 \).

Equilibrium of type P/\( \ominus \)/OS

In the second situation, the users located close to 0 adopt the P software first, those located close to 1 adopt the OS software first. Between these two populations, some users obtain a negative utility with the two software and do not adopt neither software. The user (noted \( i_{p/\ominus} \)) is defined by a null utility when adopting the P software \( U_p(i_{p/\ominus}) = 0 \). By definition of this equilibrium, we impose that \( U_{os}(i_{p/\ominus}) < 0 \) (if not, we would be in the previous situation with full adoption). Similarly, the user (noted \( i_{\ominus/OS} \)) is defined by a null utility when adopting the OS software \( U_{os}(i_{\ominus/OS}) = 0 \). Again, by definition of this equilibrium, we impose that \( U_p(i_{\ominus/OS}) < 0 \) (if not, we would be in the previous situation with full adoption). These two restrictions imply that \( 0 < i_{p/\ominus} < i_{\ominus/OS} < 1 \) (boundary conditions).

We deduce that \( i_{p/\ominus} = (v - p/\alpha) \) and \( u(2\alpha + \beta) + \beta v < \beta(2\alpha + \beta) \). From that, we deduce the expression of the profit \( \pi = p \cdot i_{p/\ominus} \) and obtain from the FOC that \( p^* = v/2 \) and \( m_p^* = v/2\alpha \). The optimal profit of the firm is then \( \pi^* = v^2/4(\alpha) \). Putting together the boundary conditions and the second-order condition, we obtain the following existing conditions for this equilibrium : \( v < 2\alpha \) and \( \beta > 2u\alpha/(2\alpha - v) \). In these conditions, we can deduce the expression of P users’ surplus \( W_p = \int_{i_{p/\ominus}}^{1} U_p(i)di = v^2/8\alpha \), of OS users’ surplus \( W_{os} = \int_{i_{\ominus/OS}}^{1} U_{os}(i)di = 2u - 3u^2/2\beta - \beta/2 \), the total surplus of all users \( W_{users} = W_p + W_{os} = (16u + v^2/\alpha - 12u^2/\beta - 4\beta)/8 \) and the total welfare \( W \) including the profit of the firm \( W = 2u + 3u^2/8\alpha - 3u^2/2\beta - \beta/2 \).

Considering the The conditions for the first type of equilibrium to occur are incompatible with those for the second type of equilibrium to occur so that we can rule out multiple equilibria.

Note: The existing conditions for equilibria of type P-OSP-OS, P-OSP-\( \ominus \)-OS and
P-OSP are all mutually exclusive. This means that starting with one parameter set, only one type of equilibrium can occur.

**Computation of the DL case**

**Proof of lemma 3:**

Suppose that the 3 platforms are active. The agent located at \( i = 0 \) clearly adopts the P platform: the utility of the P platforms users is indeed such that, for given \( p \) and \( m_{osp} \), it decreases with an increase of \( i \) more rapidly that the other utilities. If the agent located at \( i = 0 \) does not adopt the proprietary platform, this platform is not adopted and there is a contradiction with our assumptions. Let consider now the decision of the agents adjacent to the proprietary platform users. Suppose that the agent located at \( i^* \) is indifferent between using the proprietary platform and the reservation strategy. Given that \( U_{osp}(i) \) decreases with \( i \), the reservation strategy always dominates the adoption of the OSP platform for the agents with a location \( i \), such that \( i > i^* \). This case leads then to contradiction of tour assumptions and is excluded when all the platforms are active.

The same conclusion is deduced if we consider that the agent located at \( i^* \) is indifferent between the proprietary platform and the OS one. Since \( U_{os}(i) \) increases with \( i \), the OSP platform will never be adopted in this case. We deduce that if some agents adopt the OSP platform, the proprietary platform and OSP platform adopters are adjacent on the segment bearing \( i \). ■

**The P-OSP-OS case**

Let us first consider the case where all users adopt. By increasing values of \( i \), they adopt the P, the OSP and then the OS software. The user (noted \( i_{p/osp} \)) indifferent between the two software is defined by

\[
U_p(i_{p/osp}) = U_{OSP}(i_{p/osp}).
\]

Hence, \( i_{p/osp} = (am_{osp} - p)/(\alpha \lambda) \). Similarly, the user indifferent between the OSP and the OS software (noted \( i_{osp/os} \)) is defined by

\[
U_{osp}(i_{osp/os}) = U_{os}(i_{osp/os}).
\]

Thus, we obtain, \( i_{osp/os} = (u - v + \beta(\lambda - 1))/((\alpha + \beta)(\lambda - 1)) \). This case is valid only when \( 0 < i_{p/osp} < i_{osp/os} < 1 \) (boundary conditions). From this expression, we compute the expression of the profit \( \pi = p\ i_{p/osp} \) and obtain from the FOC that

\[
p^* = a(u - v + \beta(\lambda - 1))/2(\alpha + \beta)(\lambda - 1)\]

and

\[
m_p^* = a(u - v + \beta(\lambda - 1))/2(\alpha + \beta)(\lambda - 1)(a + \alpha\lambda)\].

We also deduce the number of OSP and OS users at equilibrium: \( m_{osp}^* = (v - u + \alpha(\lambda - 1))/((\alpha + \beta)(\lambda - 1)) \) and

\[
m_{os}^* = (a + 2\alpha\lambda)(u - v + \beta(\lambda - 1))/2(\alpha + \beta)(\lambda - 1)(a + \alpha\lambda) \].

From that, we can compute the profit of the firm and the surplus of all categories of users. The optimal profit of the firm is then \( \pi^* = a^2(v - u + \beta(1 - \lambda))^2/(4(\alpha + \beta)^2(\lambda - 1)^2(a + \alpha\lambda)) \). Putting together the boundary conditions and the second-order condition, we obtain the following existing conditions for this equilibrium: XXX. In these conditions, we can deduce the expression of P users’ surplus \( W_p \), of OSP users’ surplus \( W_{osp} \) of OS users’ surplus \( W_{os} \), the total surplus of all users \( W_{users} \) (with \( W_{users} = W_p + W_{osp} + W_{os} \):}

\[
W_p = \int_{0}^{i_{p/osp}} U_p(i) \, di = \ldots ;
\]

\[
W_{osp} = \int_{i_{osp/p}}^{i_{p/osp}} U_{osp}(i) \, di = \ldots ;
\]

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OSP and OS users at equilibrium:

The optimal profit of the firm is then 

\[ \pi = \text{that, we can compute the profit of the firm and the surplus of all categories of users.} \]

\[ p = \text{that} \]

As previously, the user (noted \( i_{\text{osp}} \)) is then defined by \( U_p(i_{\text{osp}}) = U_{\text{osp}}(i_{\text{osp}}) \). Hence, \( i_{\text{osp}} = (am_{\text{osp}} - p)/(\alpha \lambda) \). For values of \( i \) higher than \( i_{\text{osp}} \), users adopt the OSP platform as long as \( U_{\text{osp}}(i) > 0 \). Considering increasing values of \( i \), the 'last' user to adopt OSP (noted \( i_{\text{osp}} \)) is then defined by \( i_{\text{osp}} = 0 \). Hence, \( i_{\text{osp}} = v/(\alpha (1 - \lambda)) \). At the opposite side of the segment, users adopt the OS platform as long as \( U_s(i) > 0 \). Considering decreasing values of \( i \), the 'last' user to adopt OS (noted \( i_{\text{os}} \)) is then defined by \( i_{\text{os}} = 0 \). Thus, \( i_{\text{os}} = 1 - (u/\beta (\lambda - 1)) \).

This case is valid only when \( 0 < i_{\text{osp}} < i_{\text{os}} < 1 \) (boundary conditions). By definition, we have, \( m_p = i_{\text{osp}}, m_o = \text{OSP}, i_{\text{osp}} - i_{\text{os}}, m_o = 1 - i_{\text{os}} \). From this expression, we compute the expression of the profit \( \pi = p \cdot i_{\text{osp}} \) and obtain from the FOC that \( p^* = av/2\alpha (\lambda - 1) \) and \( m_{o}^* = av/(2\alpha (\lambda - 1)(a + \alpha \lambda)) \). We also deduce the number of OSP and OS users at equilibrium: \( m_{o^*} = \frac{v(a + 2\alpha \lambda)}{2\alpha \lambda(\lambda - 1)(a + \alpha \lambda)} \) and \( m_{o^*} = \frac{u/\beta (1 - \lambda)}{v^*(a + 2\alpha \lambda)} \). From that, we can compute the profit of the firm and the surplus of all categories of users.

The optimal profit of the firm is then \( \pi^* = a^2 v^2/(4\alpha^2 (\lambda - 1)^2(a + \alpha \lambda)) \). Putting together the boundary conditions and the second-order condition, we obtain the following existing conditions for this equilibrium: \( v < \alpha (1 - \lambda) \) and \( \beta > \frac{u\alpha}{v^* - \alpha (\lambda - 1)} \). In these conditions, we can deduce the expression of \( P \) users’ surplus \( W_p \), of OSP users’ surplus \( W_{\text{osp}} \) of OS users’ surplus \( W_{\text{os}} \), the total surplus of all users \( W_{\text{users}} \):

\[ W_p = \int_{i_{\text{osp}}}^{i_{\text{os}}} U_p(i)di = \frac{u^2 v^2 (3 - \lambda)}{2\alpha (\lambda - 1)^2(a + \alpha \lambda)} \; ; \]
\[ W_{\text{osp}} = \int_{i_{\text{osp}}}^{i_{\text{os}}} U_{\text{osp}}(i)di = \frac{v^2 (a + \alpha \lambda)^2}{2\alpha (\lambda - 1)(a + \alpha \lambda)} \; ; \]
\[ W_{\text{os}} = \int_{i_{\text{os}}}^{1} U_o(i)di = \frac{u^2}{2\alpha (1 - \lambda)} \; ; \]
\[ W_{\text{users}} = W_p + W_{\text{osp}} + W_{\text{os}} \]

The P-OSP case

In that situation, users adopt either the P platform or the OSP platform. This case is close to the first situation (P-OSP-OS). Considering increasing values of \( i \), users adopt the P first and then the OSP software. As previously, the user (noted \( i_{\text{osp}} \)) indifferent between P and OSP software is defined by \( U_p(i_{\text{osp}}) = U_{\text{osp}}(i_{\text{osp}}) \) and is characterized by \( i_{\text{osp}} = (am_{\text{osp}} - p)/(\alpha \lambda) \). Since all remaining users adopt the OSP, the user indifferent between
Figure 2: Equilibrium values (profit, surplus)
the OSP and the OS software (noted \( i_{osp/os} \), characterized by \( U_{osp}(i_{osp/os}) = U_{os}(i_{osp/os}) \) and defined by \( i_{osp/os} = (u - v + \beta(\lambda - 1))/((\alpha + \beta)(\lambda - 1)) \) as previously) is located outside the unitary segment. Then, this case is valid only if the following condition hold: \( 0 < i_{p/osp} < 1 < i_{osp/os} \) (boundary conditions). Beside, we need to check that all users get a positive utility when adopting the OSP. A sufficient condition for that is \( U_{osp}(1) > 0 \) (meaning that the user located at position 1 that get the least utility when adopting the OSP also gets a positive utility).

The expression of the profit is the same as previously \( \pi = p \cdot i_{p/osp} \) and obtain from the FOC that \( p^* = a/2 \) and \( m^*_p = a/2(a + \alpha \lambda) \), we also deduce the number of OSP users at equilibrium: \( m^*_p = 1 - m^*_p = 1 - a/2(a + \alpha \lambda) \). From that, we can compute the profit of the firm and the surplus of all categories of users. The optimal profit of the firm is then \( \pi^* = a^2/4(a + \alpha \lambda) \). Putting together the boundary conditions and the second-order condition, we obtain the following existing conditions for this equilibrium: \( u + \alpha < v + \alpha \lambda \) and \( v + \alpha \lambda > \alpha \). In these conditions, we can deduce the expression of P users’ surplus \( W_p \), of OSP users’ surplus \( W_{osp} \) of OS users’ surplus \( W_{os} \), the total surplus of all users \( W_{users} \) (with \( W_{users} = W_p + W_{osp} + W_{os} \)):

\[
W_p = \int_{i_{p/osp}}^{i_{p/osp}} U_p(i) di = \frac{a(4\alpha \lambda + a(4v + \alpha(2\lambda - 1)))}{8(a + \alpha \lambda)^2}
\]

\[
W_{osp} = \int_{i_{p/osp}}^{1} U_{osp}(i) di
\]

\[
= \frac{1}{8} \left( -\frac{4\beta(-2a + (\alpha - 2v) \beta)}{(\alpha + \beta)^2} + \frac{4(u - v)(u + v)(\alpha + 2v) \beta}{(\alpha + \beta)^2(\lambda - 1)} + \frac{4\alpha \beta^2 \lambda}{(\alpha + \beta)^2} + \frac{a^2 (a + \alpha) - a(a + 4v)}{(\alpha + \alpha \lambda)} \right)
\]

\[
W_{os} = 0
\]

The existing conditions for equilibria of type P-OSP-OS, P-OSP-\( \ominus \)-OS and P-OSP are all mutually exclusive. This means that starting with one parameter set, only one type of equilibrium can occur.

### 5.1 Comparison of the different outcomes

Total welfare (including the profit of the firm) is increasing when

\[
\alpha > 0 \& \& \left( \left( \frac{(-u + v + \beta)^2 + a(\alpha + \beta)}{\alpha + \beta} \right) > 0 \& \& (u + \alpha < v \& \& (u \geq 7\alpha + 6\beta) \| 7\alpha + 5\beta + \sqrt{33}(\alpha + \beta)^2 < 4v \| (v > 2\alpha + \beta \& \& u + 2\alpha + \beta > v \& \& 7\alpha + 5\beta + \sqrt{33}(\alpha + \beta)^2 \geq 4v \| (v < 2\alpha + \beta \& \& 2v > 2\alpha + \beta \& \& u + \frac{\sqrt{33}(\alpha + \beta)^2}{\sqrt{33}(\alpha + \beta)^2} > \beta) \| (2v < 7\alpha + 6\beta \& \& u + 2\alpha + \beta > v \& \& 7\alpha + 5\beta + \sqrt{33}(\alpha + \beta)^2 < 4v \& \& 11u + 4a + 2\sqrt{4u^2 + 4\alpha^2 + \alpha \beta - 2\beta^2 - 2v(7\alpha + 5\beta) \leq 7v + 5\beta) \| (9u^2 + v^2 + 6v \beta + \alpha \beta(-4\alpha + \beta) + 2a(\alpha + \beta) - 2a(5v^2 - 4\alpha \beta + 3\beta^2) \geq 0 \& \& 7v + 5\beta + 2\sqrt{4u^2 + 4\alpha^2 + \alpha \beta - 2\beta^2 - 2v(7\alpha + 5\beta) > 11u + 4a \& \&
\right)
\]
Welfare of P-users is increasing when (same conditions as previously):

\[
\alpha > 0 \land \frac{-(u + v + \beta)^2 + u(a + \beta)}{\alpha + \beta} > 0 \land ((u + \alpha < v) \land \\
\left( \begin{array}{c}
\frac{(\alpha + \beta)^2(4v^2 - 12\alpha + 3\alpha^2 - 8\alpha\beta + 2\alpha^3)}{(3\alpha + 2\beta)^2} < u \land \\
4v \geq 11a + 8\beta \quad \left( 3\alpha + 2\beta + \sqrt{6\alpha^2 + 10\alpha\beta + 4\beta^2} < 2v \right) \Rightarrow
\end{array} \right)
\]

\[
\left( v > 2\alpha + \beta \land \& u + 2\alpha + \beta > v \land \& 3\alpha + 2\beta + \sqrt{6\alpha^2 + 10\alpha\beta + 4\beta^2} < 2v \right) \Rightarrow
\]

\[
\left( u + 2\alpha + \beta > v \land \& u + \frac{(\alpha + \beta)^2(4v^2 - 12\alpha + 3\alpha^2 - 8\alpha\beta + 2\alpha^3)}{(3\alpha + 2\beta)^2} < \beta + \frac{\sqrt{\alpha + \beta}}{3\alpha + 2\beta} \land \\
3\alpha + 2\beta + \sqrt{6\alpha^2 + 10\alpha\beta + 4\beta^2} = 2v
\right)
\]

Welfare all users is increasing when:

\[
\alpha > 0 \land \frac{-(u + v + \beta)^2 + u(a + \beta)}{\alpha + \beta} > 0 \land ((u + \alpha < v) \land \\
\left( \begin{array}{c}
7v + 5\beta + 2\sqrt{4v^2 + 4\alpha^2 + \alpha\beta - 2\beta^2 - 2v(7\alpha + 5\beta)} \leq 11u + 4a \land \\
2v \geq 7\alpha + 6\beta \quad \left( 7\alpha + 5\beta + \sqrt{33}(\alpha + \beta)^2 < 4v \right) \Rightarrow
\end{array} \right)
\]

\[
\& (v < 2\alpha + \beta \land \& 2v > 2\alpha + \beta \land \& u + \frac{\sqrt{\alpha + \beta}}{3\alpha + 2\beta} > \beta)
\]

\[
\left( u < 2\alpha + \beta \land \& u + 2\alpha + \beta > v \land \& 7\alpha + 5\beta + \sqrt{33}(\alpha + \beta)^2 < 4v \land \\
\left( 2v < 7\alpha + 6\beta \land \& u + 2\alpha + \beta > v \land \& 7\alpha + 5\beta + \sqrt{33}(\alpha + \beta)^2 < 4v \land \\
11u + 4a + 2\sqrt{4v^2 + 4\alpha^2 + \alpha\beta - 2\beta^2 - 2v(7\alpha + 5\beta)} \leq 7v + 5\beta
\right)
\]

\[
\left( 2v < 7\alpha + 6\beta \land \& u + 2\alpha + \beta > v \land \& 7\alpha + 5\beta + \sqrt{33}(\alpha + \beta)^2 < 4v \land \\
11u + 4a + 2\sqrt{4v^2 + 4\alpha^2 + \alpha\beta - 2\beta^2 - 2v(7\alpha + 5\beta)} > 7v + 5\beta \land \& \left( \sqrt{5\beta} > 5\alpha + \beta \left( \sqrt{2a} > \beta \land \& \beta > 0 \right) \left( \sqrt{2a} < \beta \land \& 5\alpha + \beta > \sqrt{6}\beta \right) \right)
\right)
\]